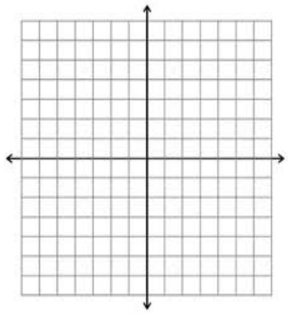
Precalc Warm Up # 6-2

Graph and state the domain and range

1. $y = x^2 - 10x + 28, \; x>4$

2. $x^2 + y^2 - 6x + 4y + 11 = 0$



3. $f(x) = 2x + 1 \quad g(x) = x^2$ find (and state domain):

a. $f(g(x))$

b. $g(f(x))$

EXERCISES 5.4.1 p. 154 HW Questions?

3. All of the following functions are mappings of $\mathbb{R} \mapsto \mathbb{R}$ unless otherwise stated.

(a) Determine the composite functions $(f \circ g)(x)$ and $(g \circ f)(x)$, if they exist.

(b) For the composite functions in (a) that do exist, find their range.

ii. $f(x) = x^2 + 1, g(x) = \sqrt{x}, x \geq 0$

iv. $f(x) = \frac{1}{x}, x \neq 0, g(x) = \frac{1}{x}, x \neq 0$

vi. $f(x) = x^2 - 1, g(x) = \frac{1}{x}, x \neq 0$

viii. $f(x) = x - 4, g(x) = |x|$

x. $f(x) = \sqrt{4-x}, x \leq 4, g(x) = x^2$

xii. $f(x) = x^2 + x + 1, g(x) = |x|$

xii) a) $f(g(x)) \rightarrow r_g \overset{?}{\subseteq} d_f$

$[0, \infty) \subseteq \mathbb{R}$ ✓

$f(|x|) = (|x|)^2 + |x| + 1$

$f(g(x)) = x^2 + |x| + 1$

$g(f(x)) \rightarrow r_f \subseteq d_g$

$[\frac{3}{4}, \infty) \subseteq \mathbb{R}$ ✓

$d_f = \mathbb{R}$

$r_f = [\frac{3}{4}, \infty)$

$d_g = \mathbb{R}$

$r_g = [0, \infty)$

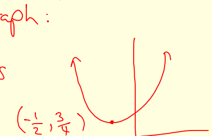
$f \rightarrow$ vertex: $(-\frac{1}{2}, \frac{3}{4})$

$y = (-\frac{1}{2})^2 - \frac{1}{2} + 1 = \frac{3}{4}$

$g(x^2 + x + 1)$

$= |x^2 + x + 1|$ *

* Think of this as all the outcomes of the parabola are positive. Look at the graph:



All outcomes are already positive!

So: $g(f(x)) = x^2 + x + 1$

b) range: $[0, \infty)$

a) $g(f(x)) \rightarrow r_f \overset{?}{\subseteq} d_g$

$[\frac{3}{4}, \infty) \subseteq \mathbb{R}$ ✓

3. All of the following functions are mappings of $\mathbb{R} \mapsto \mathbb{R}$ unless otherwise stated.

(a) Determine the composite functions $(f \circ g)(x)$ and $(g \circ f)(x)$, if they exist.

(b) For the composite functions in (a) that do exist, find their range.

$d_f: x \neq -1$

$r_f: y \neq 0$

$\mathbb{R}$ $\mathbb{R}$

xiv. $f(x) = \frac{1}{x+1}, x \neq -1, g(x) = x - 1$

$r_g \subseteq d_f$

$\mathbb{R} \subseteq x \neq -1$

$f(g(x))$ DNE

$g(f(x))$

$r_f \subseteq d_g$ ✓

$\neq 0$ $\mathbb{R}$

xvi. $f(x) = 4^x, g(x) = \sqrt{x}$

$d_f = \mathbb{R}$ $d_g = [0, \infty)$

$r_f \Rightarrow (0, \infty)$ $r_g = [0, \infty)$

$f(g(x)) \rightarrow r_g \subseteq d_f$

$[0, \infty) \subseteq \mathbb{R}$ ✓

$f(\sqrt{x}) = 4^{\sqrt{x}}$

$g(f(x)) = g(4^x)$ $\longrightarrow$ $r_f \overset{?}{\subseteq} d_g$

$(0, \infty) \subseteq [0, \infty)$

$\boxed{= \sqrt{4^x}}$

$= (4^x)^{\frac{1}{2}}$

$= (4^{\frac{1}{2}})^x \longrightarrow 4^{0.5x}$!!

$\boxed{= 2^x}$

4. Given the functions $f: x \mapsto 2x + 1, x \in]-\infty, \infty[$ and $g: x \mapsto x + 1, x \in]-\infty, \infty[$

Find the functions (a) $(f \circ g)$ (b) $(g \circ f)$ (c) $(f \circ f)$

$r_f \overset{?}{\subseteq} d_f$ $2(2x+1) + 1$

$\mathbb{R}$ $\mathbb{R}$

5. Given that $f: x \mapsto x + 1, x \in \mathbb{R}$ and $g \circ f: x \mapsto x^2 + 2x + 2, x \in \mathbb{R}$, determine the function g.

$f(x) = x + 1$ $g(f(x)) = x^2 + 2x + 2$

$g(x+1)$

I want a g function that will let me input $f(x) = x+1$

So complete the square:

$g(f(x)) = x^2 + 2x \underline{+1} + 2 \underline{-1}$

$g(f(x)) = (\underbrace{x+1}_{})^2 + 1$

this is my input $f(x)$

So $g(x) = x^2 + 1$

7. If $g: x \mapsto x^3 + 1, x \in \mathbb{R}$ and $f: x \mapsto \sqrt{x}, x \in [0, \infty[$, evaluate (a) $(g \circ f)(4)$ ~~(b)~~ $(f \circ g)(2)$

$f(g(x))$ DNE

$r_g \subseteq d_f$

$\mathbb{R} \not\subseteq [0,\infty)$

$f(g(x)) = 0$

9. Solve the equation $(f \circ g)(x) = 0$, where

(a) $f: x \mapsto x + 5, x \in \mathbb{R}$ and $g: x \mapsto x^2 - 6, x \in \mathbb{R}$.

(b) $f: x \mapsto x^2 - 4, x \in \mathbb{R}$ and $g: x \mapsto x + 1, x \in \mathbb{R}$.

a) $f(x) = x + 5 \quad g(x) = x^2 - 6$

$f(g(x)) = f(x^2 - 6)$

$f(g(x)) = x^2 - 6 + 5$

$f(g(x)) = x^2 - 1$

$0 = x^2 - 1$

$x = \pm 1$

b) $f(x) = x^2 - 4 \quad g(x) = x + 1$

$f(g(x)) = f(x + 1)$

$f(g(x)) = (x+1)^2 - 4$

$f(g(x)) = x^2 + 2x + 1 - 4$

$0 = x^2 + 2x - 3$

$0 = (\quad x \quad)$

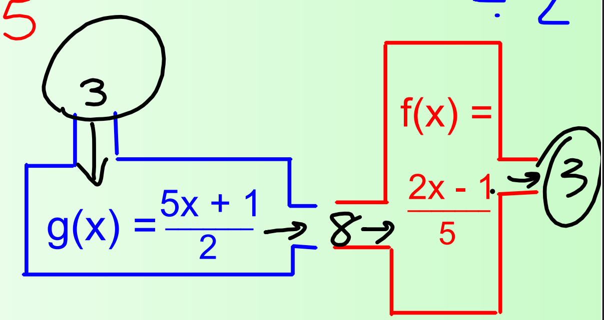
$f(x) = \dfrac{2x - 1}{5}$

input
× 2
− 1
÷ 5

$g(x) = \dfrac{5x + 1}{2}$

input
× 5
+ 1
÷ 2

1. Find f(g(3))



2. Find $f(g(x)) = x$

Find f^{-1} for the following. State domain and range for function and inverse.

1. $f(x) = 4x - 5$

input
$\times 4$
-5

$f^{-1} \rightarrow$ input
$+5$
$\div 4$

$$f^{-1}(x) = \frac{x+5}{4}$$

2. $f(x) = 2x^3 - 1$

input
cube
$\times 2$
-1

f^{-1}
input (x)
$+1$
$\div 2$
$\sqrt[3]{\ }$

$$f^{-1}(x) = \sqrt[3]{\frac{x+1}{2}}$$

Check:

$$f(f^{-1}(x)) = f\left(\sqrt[3]{\frac{x+1}{2}}\right)$$

$$= 2\left(\sqrt[3]{\frac{x+1}{2}}\right)^3 - 1$$

$$= \cancel{2}\left(\frac{x+1}{\cancel{2}}\right) - 1$$

$$= x + 1 - 1$$

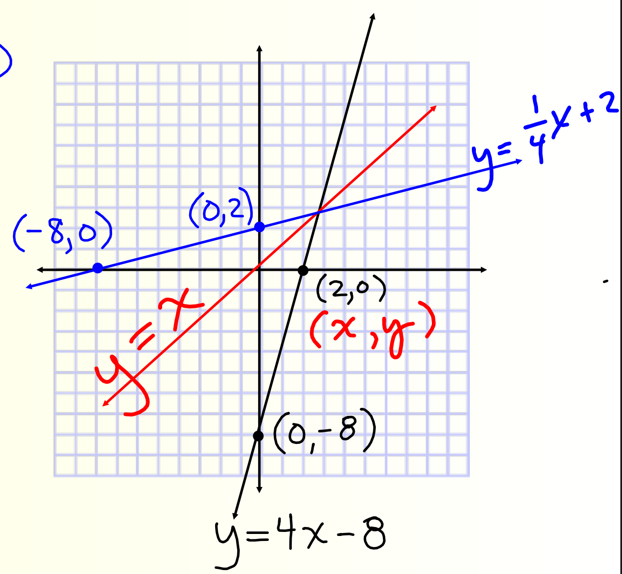
$$= x$$

How are the graphs of two inverse function related?

Graph $f(x) = 4x - 8$ and it's inverse on the same axis

input
$\times 4$
-8

inverse
input
$+8$
$\div 4$



$$f^{-1}(x) = \frac{x+8}{4}$$

$$f^{-1}(x) = \frac{1}{4}x + 2$$

The graphs of f(x) and $f^{-1}(x)$ are reflection images of one another, over the line $\boxed{y = x}$

Graph $f(x) = \sqrt{x-4}$ and its inverse on the same axis

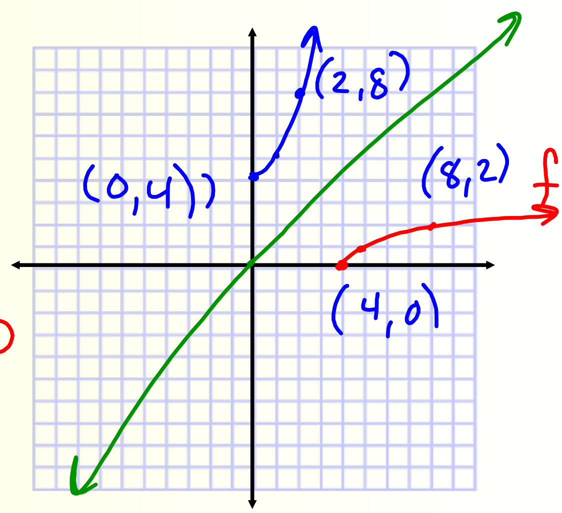
input
- 4
√

dom: $x \geq 4$
range:
$y \geq 0$

$\underline{\text{input}}$
$(\ )^2 + 4$

$f^{-1}(x) = x^2 + 4$

dom: → $x \geq 0$
range:
$y \geq 4$

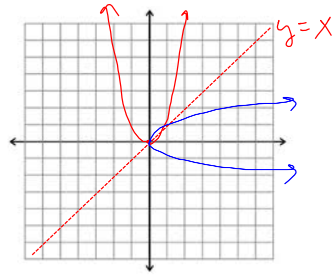


Many functions don't have inverses that are functions.
For example:

$f(x) = x^2$

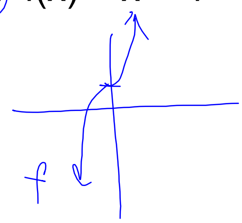
$\pm\sqrt{x} = \sqrt{y^2}$

$\boxed{y = \pm\sqrt{x}}$



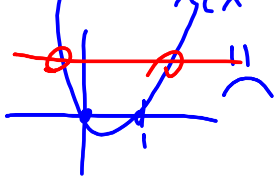
Its inverse is not a function. In order for a function to have an inverse function, the original function must be one to one. Which of the following functions have inverse functions?

1. $f(x) = x^3 + 1$

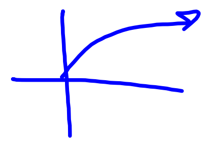


2. $g(x) = x^2 - x$

$x(x-1)$



3. $h(x) = \sqrt{x}$



Remember:

To be a one to one function, it must pass both vertical and horizontal line tests.

For the inverse function of f (called f^{-1}) to exist, then

f inverse

1. f must be a one to one function

2. The domain of f is the range of f^{-1}

   The range of f is the domain of f^{-1}

If the above holds true,

and $f(f^{-1}(x)) = f^{-1}(f(x)) = x$,

then $f(x)$ and $f^{-1}(x)$ are inverses of each other.

Steps to find the inverse function of another function:

Step 1: Check that your function is one to one. If it is not, the inverse will exist, but not as a function.

Step 2: Find the domain and range of your function. Switch them and set them as your domain and range for your inverse.

Step 3: In your equation, swap x for y. Then solve for y. This is your inverse!

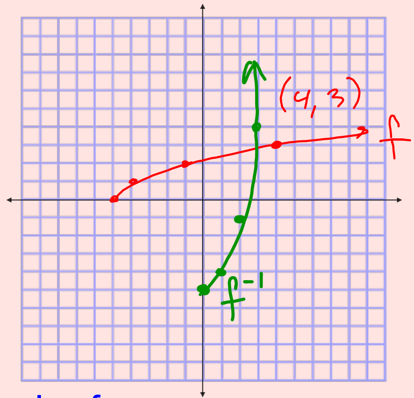
Find the inverse function of $f(x) = \sqrt{x+5}$ and graph both functions.

$f^{-1}(x) = x^2 - 5$

$d: x \geq 0$

$r: y \geq -5$

$y = \sqrt{x+5}$



Inverse: swap x & y and solve for y

$(x)^2 = (\sqrt{y+5})^2$

$x^2 = y + 5$

$\boxed{x^2 - 5 = y}$

Find the inverse function of f(x)= x²+4x , x < -2 $(-2, -4)$

State dom and range and graph both $= x(x+4)$ $r_f = (-4, \infty)$

$y = x^2 + 4x$

Inv

$x = \underline{y^2 + 4y + 4} - 4$

$x = (y+2)^2 - 4$

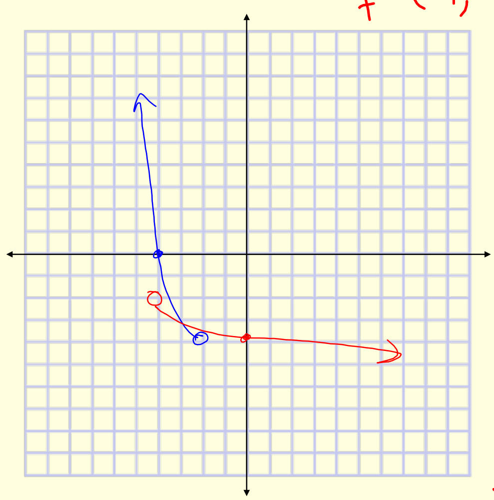
$\sqrt{x+4} = \sqrt{(y+2)^2}$

$\pm\sqrt{x+4} = y + 2$

$y =$

$y = -2 - \sqrt{x+4}$

$f^{-1}(x) = -2 - \sqrt{x+4}; \; x > -4$



$d_{f^{-1}}: (-4, \infty)$

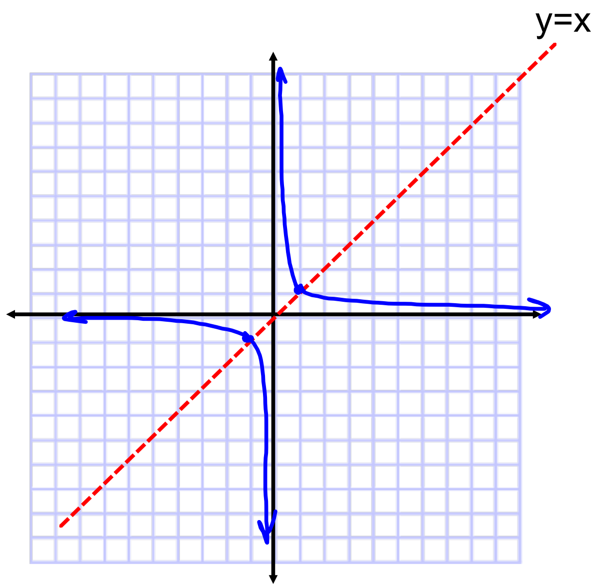
$r_{f^{-1}}: (-\infty, -2)$

Can you think of any function that is it's own inverse?

$y = \frac{1}{x}$

also

$y = -x$



Find the inverse of $f(x) = \frac{2}{x-1} + 1$, x>1

x	y
2	3
3	2

Inverse:

$x = \frac{2}{y-1} + 1$

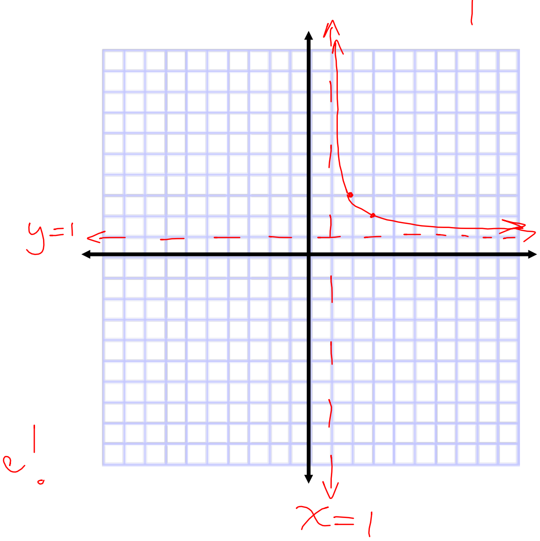
$\frac{x-1}{1} = \frac{2}{y-1}$

interchange extremes:

$y - 1 = \frac{2}{x-1}$

$f^{-1}(x) = \frac{2}{x-1} + 1$

f is its own inverse!



Group Event Wed:

Find the relationship that represents all points equidistant from 2 given points.

Using point slope form, midpoint, distance formula

Write g in terms of f, and describe transformations.

Determine if a composite exists.

Group Event

* You can only write on your own paper

* Work together and make sure everyone agrees on what each person is writing on their paper. Check details!

* One paper will be selected at random for grading.

HW: SL book

p. 164 #1 LC , 3, 5abcd,

and 9 -12

Bring PC book
Wed - Th